DATA ACQUISITION AND NYQUIST SAMPLING THEOREM

ME 4231
Rajesh Rajamani
Department of Mechanical Engineering
University of Minnesota

PC Based Control System

Loop
{
Read voltages from sensors

Compute voltage to be sent to actuator

Send voltage to actuator
}

PC Based Control System

Example: Automotive Cruise Control

Loop

{

Read voltages from wheel speed sensors, determine speed of vehicle

Compute difference between desired speed and actual speed

•
•
•

}

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PC Based Control System

Example: Automotive Cruise Control

Loop

{

•
•
•

Compute whether throttle angle should be increased or decreased and by how much

Compute voltage to be sent to throttle actuator

Send voltage to throttle actuator

}

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DATA ACQUISITION CARD

Common Tasks

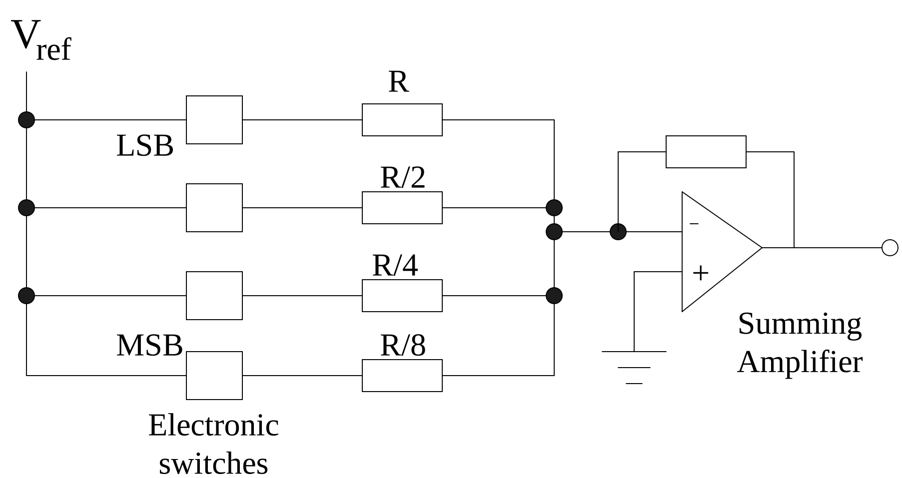
- Read voltages
  - a2d (analog to digital conversion)
  - digital inputs

    The signal from a sensor can be analog or digital
- Send out voltages
  - d2a (digital to analog conversion)
  - digital outputs

    The voltage to be sent to an actuator can be analog or digital

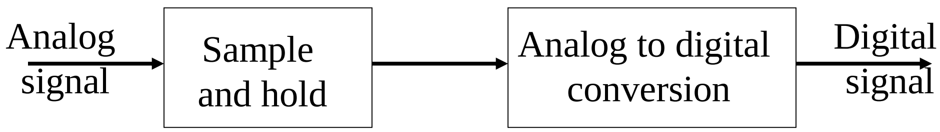
DATA ACQUISITION

- Weighted-resistor D2A (digital 2 analog conversion)



DATA ACQUISTION

- Analog-to-digital conversion



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DIGITIZATION

- Two types of digitization
  - Digitization in time, called "sampling"
    - Depends on speed and complexity of real-time program
    - Depends on speed of data acquisition card
  - Digitization in value
    - Depends on resolution of data acquisition card (12 bit, 16 bit, etc)

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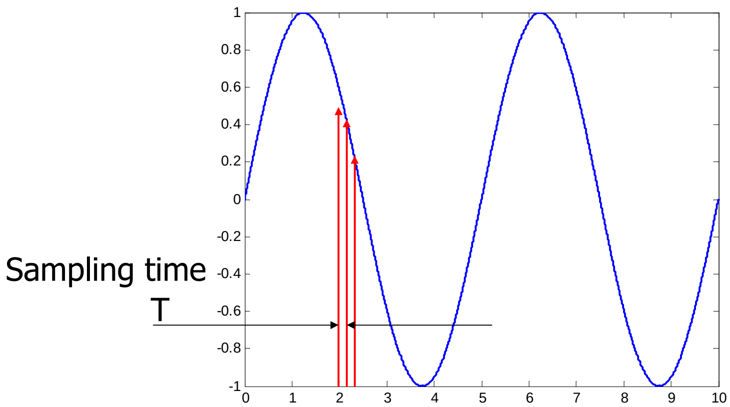
DIGITIZATION

- Digitization in time (“sampling”)



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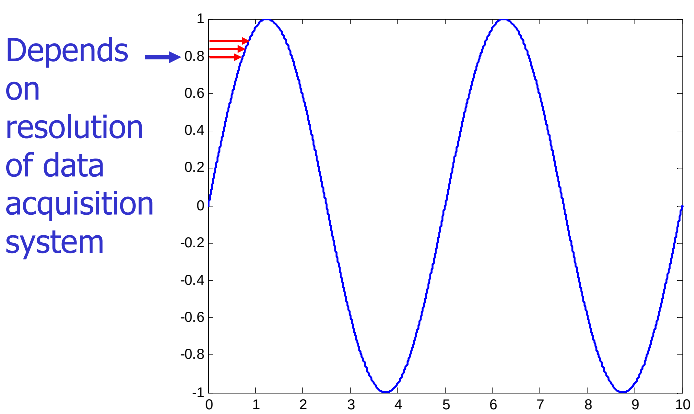
DIGITIZATION

- Digitization in value



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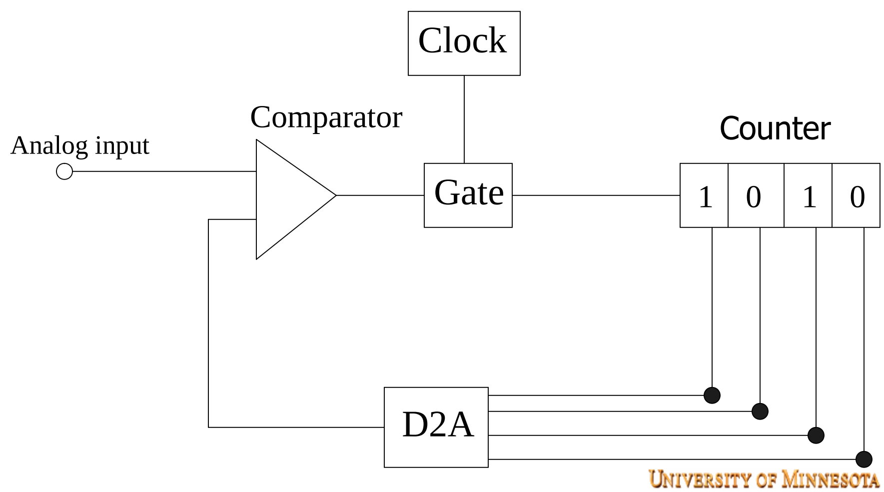
DATA ACQUISITION

- Successive Approximation A2D



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DATA ACQUISITION

- Data acquisition card from Sensoray in lab
  - Model 626 PCI Multi-function I/O board
  - 16 differential analog inputs (16-bit)
  - 4 analog outputs (14-bit)
  - 48 digital I/O channels
  - 6 24 bit up/down counters

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SAMPLING THEOREM

The **sampling theorem** states that for a limited bandwidth (band-limited) signal with maximum frequency f_{max}, the equally spaced sampling frequency f_s must be greater than twice of the maximum frequency f_{max}, i.e.,

$$f_s > 2 \cdot f_{max}$$

in order to have the signal be uniquely reconstructed without **aliasing**.

The frequency $2 \cdot f_{max}$ is called the **Nyquist sampling rate**.

Sampling theorem articulated by Nyquist in 1928

Mathematically proved by Shannon in 1949.

Some books use the term "Nyquist Sampling Theorem", and others use "Shannon Sampling Theorem".

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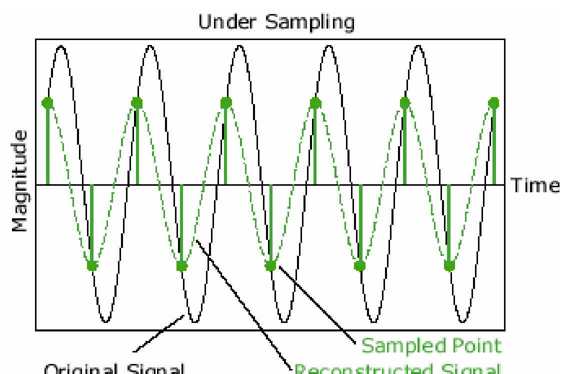
Under Sampling

When the sampling rate is lower than or equal to the **Nyquist rate**, a condition defined as **under sampling**, it is impossible to rebuild the original signal according to the **sampling theorem**.



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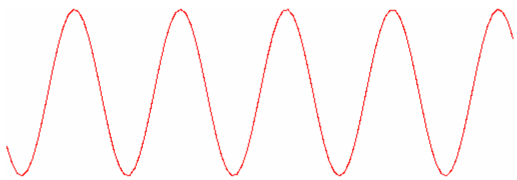
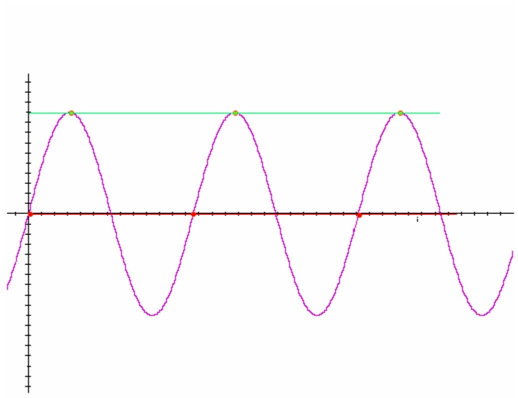
ALIASING

Suppose we are sampling a sine wave. How often do we need to sample it to figure out its frequency?



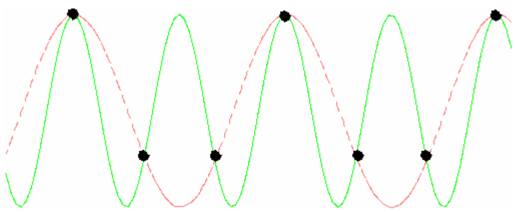
ALIASING

If we sample at 1 time per cycle, we can think it's a constant



ALIASING

If we sample at 1.5 times per cycle, we can think it's a lower frequency sine wave



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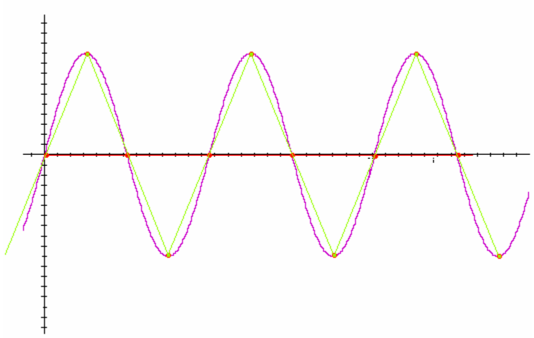
ALIASING

If we sample at twice the maximum frequency, i.e Nyquist Rate, we start to make some progress. In this case we see we get a sawtooth wave that begins to start crudely approximating a sine wave



However, phase mismatches will distort the signal.

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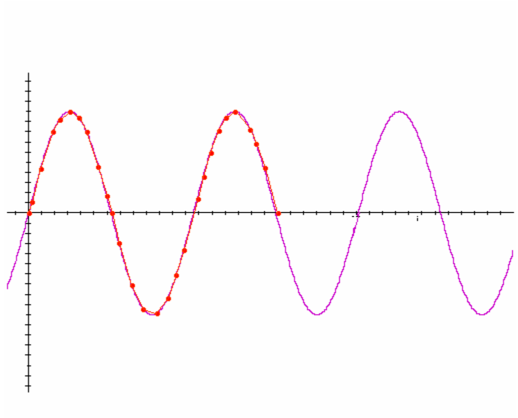
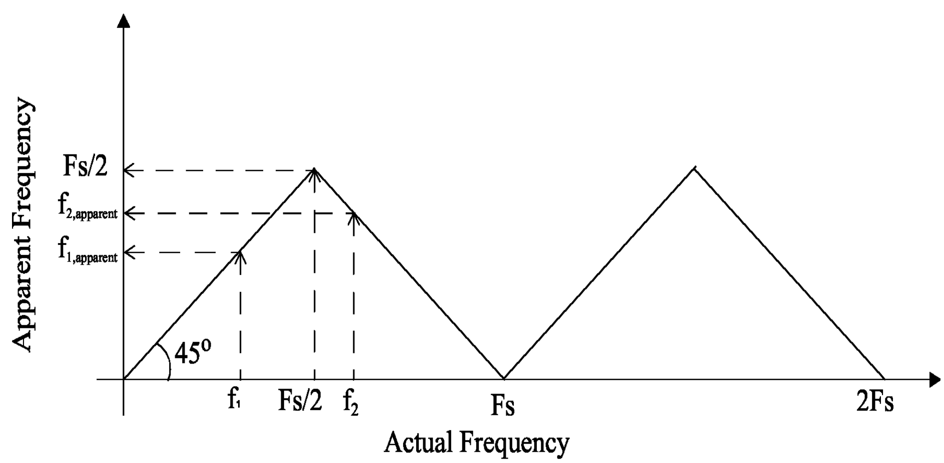
ALIASING

Sampling at many times per cycle



For loss-less digitization, the sampling rate should be *at least twice* the maximum frequency responses. Indeed many times more the better.

ALIASING



ANTI-ALIAS FILTERS

Therefore, an analog filter is typically applied before sampling to ensure that no components with frequencies greater than half the sample frequency remain. This is called an anti-aliasing filter.

The quality of analog-to-digital-converters (A/D-Converters) depends critically upon that filter, since a poor filter causes phase distortion and other difficulties.

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A2D Conversion

1	0	· · ·	0	1

n bit binary number

$2^n - 1$ possible combinations, other than zero

e.g.

1	0

3 possible combinations
 - 3 non-zero numbers can be represented

n bit binary number

1	0	· · ·	0	1

sign bit

$2^{n-1} - 1$ positive numbers

2^{n-1} negative numbers can be represented

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A2D Conversion

For a 16-bit a2d, set to operate between -10 to 10 V, what voltages do 15,000 and 15,001 correspond to ?

$$2^{n-1} - 1 = 32767$$

Hence numbers from -32768 to 32767 can be represented

$$15{,}000: \quad \frac{V}{V_{max}} = \frac{15{,}000}{32{,}767} \Rightarrow V = \frac{15{,}000}{32{,}767} \text{ x } 10 = 4.5778V$$

$$15{,}001: \quad \frac{V}{V_{max}} = \frac{15{,}001}{32{,}767} \Rightarrow V = \frac{15{,}001}{32{,}767} \text{ x } 10 = 4.5781V$$

z : $V = \frac{z}{32{,}767} \text{ x } 10V$ for $z > 0$ $\quad V = \frac{z}{32{,}768} \text{ x } 10V$ for $z < 0$

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D2A Conversion

For a 14-bit d2a, set to operate between -10 to 10 V, what voltages do 4,000 and 4,001 correspond to ?

$$2^{n-1} - 1 = 8191$$

Hence numbers from -8192 to 8191 can be represented

$$4{,}000: \quad \frac{V}{V_{max}} = \frac{4{,}000}{8{,}191} \Rightarrow V = \frac{4{,}000}{8{,}191} \text{ x } 10 = 4.8834V$$

$$4{,}001: \quad \frac{V}{V_{max}} = \frac{4{,}001}{8{,}191} \Rightarrow V = \frac{4{,}001}{8{,}191} \text{ x } 10 = 4.8846V$$

z : $V = \frac{z}{8{,}191} \text{ x } 10V$ for $z > 0$ $\quad V = \frac{z}{8{,}192} \text{ x } 10V$ for $z < 0$

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PC Based Control System

Loop
{
Read voltages from sensors

Compute voltage to be sent to actuator

Send voltage to actuator

Wait until sampling time has been reached
}

Lab 4

- Task 1
- Demonstration of the A2D Converter
  - Set the power supply to different voltages
  - Check and see if your a2d program can read those voltages correctly
  - Change the voltage range settings in your program and repeat

Lab 4

- Task 2
- Demonstration of the D2A Converter
  - Send out different voltages from channel 0 by writing a program that takes a user input from the screen
  - The user enters an integer number, e.g. 6000
  - Check the output voltage on a multimeter to see if your program works correctly

Lab 4

- Task 3
- Demonstration of Aliasing with the A2D Converter
  - Set the sampling frequency to be 1000 Hz
  - Change input signal frequency on the function generator to vary from 50Hz to 1500 Hz according to the given table
  - For each input frequency write down the estimated frequency of the output signal from the oscilloscope.